

Effects in a pi - *using session types as an effect system*

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CCS



Functions as processes, Milner (1992)



λ -calculus

π -calculus

Church (1930s)

A Calculus of Mobile Processes (part 1), (1992)

Milner, Parrow, Walker

UI



*Functions as session-typed processes,
Tohninho, Caires, Pfenning (2012)*



UI



what

simple types



what & how

session types

Church (1940)

*Language primitives and type disciplines for
structured communication-based programming*

Honda, Vasconcelos, Kubo (1998)

+



work in progress



how

effect systems

Integrating functional and imperative programs

Gifford, Lucassen (1986)

This talk

π -calculus recap

$c!\langle V \rangle.P$	send V on c then act as P
$c?(x).P$	receive on c , bind to x , then act as P
$P \mid Q$	do P and Q in parallel
$\nu c (P)$	channel creation/binding [restriction]
$! P$	process replication
0	inactive process

$$(c?(x).P \mid c!\langle V \rangle.Q) \rightarrow (P[V/x] \mid Q) \quad (\beta \text{ reduction})$$

(commutativity)

$$P \mid Q = Q \mid P$$

(associativity)

$$P \mid (Q \mid R) = (P \mid Q) \mid R$$

(scope extrusion)

$$\nu c (P \mid Q) = \nu c (P) \mid Q \quad (\text{if } c \# Q)$$

Session primitives primer

$c \triangleright \{L_1 : P_1, \dots, L_n : P_n\}$


offer n choices

$c \triangleleft L_i . P$

select label i then act as P

$(c \triangleright \{L_1 : P_1, \dots, L_n : P_n\} \mid \underline{c} \triangleleft L_i . Q)$ (β reduction)

$\rightarrow (P_i \mid Q)$



dual end-point

$(c?(x).P \mid \underline{c}!\langle V \rangle.Q)$ (β reduction)



dual end-point

Session types primer

$$\Gamma ; \Delta \vdash P$$

value environment

$$x_1 : A_1, \dots, x_n : A_n$$

session environment

$$c_1 : S_1, \dots, c_n : S_n$$

$$\text{(recv)} \frac{\Gamma, x : A ; \Delta, c : S \vdash P}{\Gamma ; \Delta, c : ?[A].S \vdash c?(x).P}$$

$$\text{(send)} \frac{\Gamma ; \Delta, c : S \vdash P \quad \Gamma ; . \vdash V : A}{\Gamma ; \Delta, c : ![A].S \vdash c!\langle V \rangle.P}$$

Session types primer (2)

dual session type

$$\text{(inact)} \quad \Gamma; c : \mathbf{end} \vdash \mathbf{0} \qquad \text{(restr)} \quad \frac{\Gamma; \Delta, c : S, \underline{c} : \underline{S} \vdash P}{\Gamma; \Delta \vdash \nu c.P}$$

Duality: *ensures absence of communication errors*

$$\underline{?[A].S} = ![A].\underline{S} \qquad \underline{![A].S} = ?[A].\underline{S} \qquad \underline{\mathbf{end}} = \mathbf{end}$$

e.g. ~~$c!\langle 0 \rangle.c!\langle 1 \rangle \mid \underline{c?}(x)$~~ ~~$c!\langle 0 \rangle \mid \underline{c?}(x).c?(y) \mid c!\langle 1 \rangle$~~

$$\text{(par)} \quad \frac{\Gamma; \Delta_1 \vdash P \qquad \Gamma; \Delta_2 \vdash Q}{\Gamma; \Delta_1 \odot \Delta_2 \vdash P \mid Q}$$

Session types primer (2)

dual session type

$$\text{(inact)} \quad \Gamma; c : \mathbf{end} \vdash \mathbf{0} \qquad \text{(restr)} \quad \frac{\Gamma; \Delta, c : S, \underline{c} : \underline{S} \vdash P}{\Gamma; \Delta \vdash \nu c.P}$$

Duality: *ensures absence of communication errors*

$$\underline{?[A].S} = ![A].\underline{S} \qquad \underline{![A].S} = ?[A].\underline{S} \qquad \underline{\mathbf{end}} = \mathbf{end}$$

e.g. ~~$c!\langle 0 \rangle.c!\langle 1 \rangle \mid \underline{c?}(x)$~~ $c!\langle 0 \rangle \mid \underline{c?}(x).d?(y) \mid d!\langle 1 \rangle$ ✓

$$\text{(par)} \quad \frac{\Gamma; \Delta_1 \vdash P \qquad \Gamma; \Delta_2 \vdash Q}{\Gamma; \Delta_1 \odot \Delta_2 \vdash P \mid Q}$$

Session types primer (3)

$$\text{(branch)} \frac{\Gamma; \Delta, c : S_0 \vdash P_0 \quad \dots \quad \Gamma; \Delta, c : S_n \vdash P_n}{\Gamma; \Delta, c : \&[l_0 : S_0 \dots l_n : S_n] \vdash c \triangleright \{l_0 : P_0 \dots l_n : P_n\}}$$

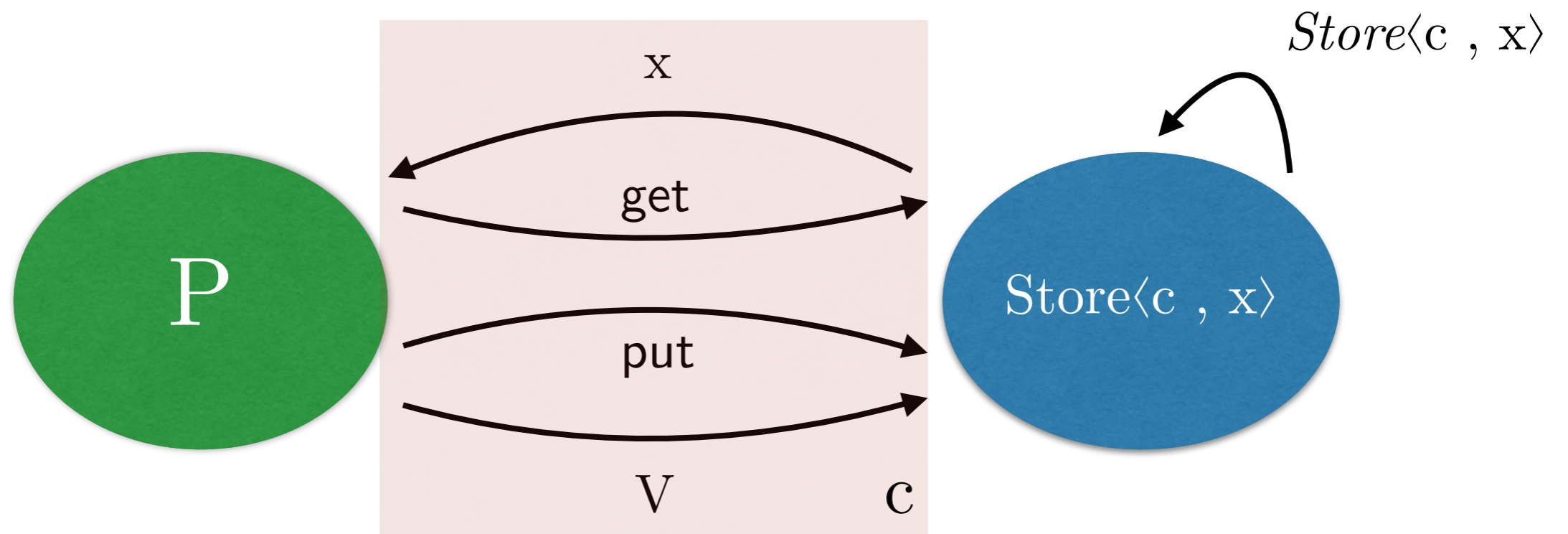
$$\text{(select)} \frac{\Gamma; \Delta, c : S \vdash P}{\Gamma; \Delta, c : \oplus[l : S] \vdash c \triangleleft l . P}$$

Duality:

$$\underline{\&[l_0 : S_0 \dots l_n : S_n]} = \oplus[l_0 : \underline{S_0} \dots l_n : \underline{S_n}]$$

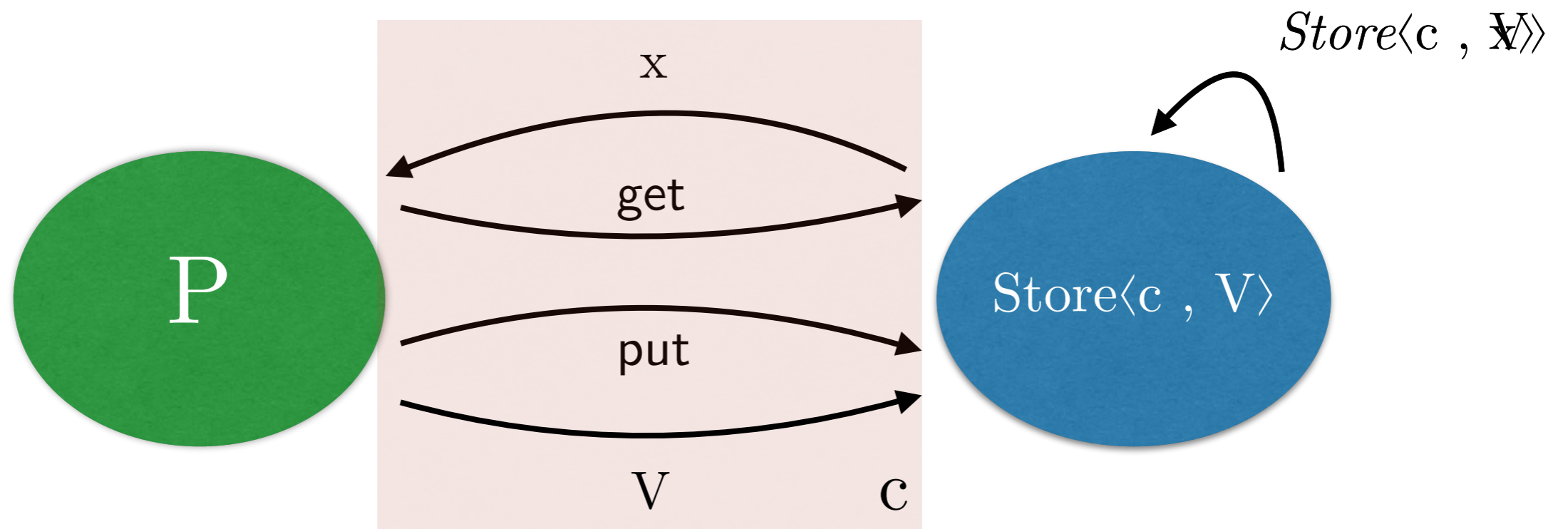
$$\underline{\oplus[l_0 : S_0 \dots l_n : S_n]} = \&[l_0 : \underline{S_0} \dots l_n : \underline{S_n}]$$

Effects in a π : *Variable agent*



Effects in a π : *Variable agent*

```
def Store(c, x) = c ▷ {get : c!⟨x⟩.Store⟨c, x⟩,  
put : c?(y).Store⟨c, y⟩,  
stop : 0}           in Store⟨c , i⟩
```



Effects in a π : *Variable agent*

$\text{def } Store(c, x) = c \triangleright \{ \text{get} : c! \langle x \rangle . Store \langle c, x \rangle ,$ Server
 $\text{put} : c? \langle y \rangle . Store \langle c, y \rangle ,$
 $\text{stop} : \mathbf{0} \}$ in $Store \langle c, i \rangle$

$\text{get}(c)(x).P = (c \triangleleft \text{get}).c? \langle x \rangle .P$
 $\text{put}(c) \langle V \rangle .P = (c \triangleleft \text{put}).c! \langle V \rangle .P$
 $\text{stop} = (c \triangleleft \text{stop}).\mathbf{0}$

$c : \oplus[\text{get} : ?[A]. S]$

$c : \oplus[\text{put} : ![A]. S]$

session types

Client

e.g. increment store

$(\text{get}(c)(x).\text{put}(c) \langle x+1 \rangle .\mathbf{0} \mid Store \langle c, i \rangle)$

$c : \oplus[\text{get} : ?[Z]. \oplus[\text{put} : ![Z].\text{end}]]$ \vdash $\text{get}(c)(x).\text{put}(c) \langle x+1 \rangle .\mathbf{0}$

describes effect interaction

$$\Gamma \vdash M : \tau$$

Effect system

monoid $(\mathbf{F}, \bullet, \emptyset)$

$$\begin{array}{c}
 \text{abs} \frac{\Gamma, x : \sigma \vdash M : \tau}{\Gamma \vdash \lambda x.M : \sigma \rightarrow \tau} \quad \text{app} \frac{\Gamma \vdash M : \sigma \rightarrow \tau \quad \Gamma \vdash N : \sigma}{\Gamma \vdash M N : \tau} \quad \text{var} \frac{x : \sigma \in \Gamma}{\Gamma \vdash x : \sigma} \\
 \\
 \text{bind} \frac{\Gamma \vdash M : \mathbf{T} \mathbf{F} \sigma \quad \Gamma, x : \sigma \vdash N : \mathbf{T} \mathbf{G} \tau}{\Gamma \vdash \text{let } x \Leftarrow M \text{ in } N : \mathbf{T} (\mathbf{F} \bullet \mathbf{G}) \tau} \quad \text{return} \frac{\Gamma \vdash M : \tau}{\Gamma \vdash \langle M \rangle : \mathbf{T} \emptyset \tau}
 \end{array}$$

The marriage of effects and monads, Wadler & Thiemann (1992)

Analysis style [Gifford, Lucassen (1986), etc.]

$$\frac{\Gamma, x : \sigma \vdash M : \tau, \mathbf{F}}{\Gamma \vdash \lambda x.M : \sigma \xrightarrow{\mathbf{F}} \tau, \emptyset} \quad \frac{\Gamma \vdash M : \sigma \xrightarrow{\mathbf{H}} \tau, \mathbf{F} \quad \Gamma \vdash N : \sigma, \mathbf{G}}{\Gamma \vdash M N : \tau, \mathbf{F} \bullet \mathbf{G} \bullet \mathbf{H}}$$

Effect system for state

Effect monoid: $(\text{List } \{\text{put } t, \text{get } t \mid t \in \tau\}, ++, [])$

Effect operations:
 $\text{put} : \tau \rightarrow \mathbf{T} [\text{put } \tau] ()$
 $\text{get} : \mathbf{T} [\text{get } \tau] \tau$

e.g. increment store

$$\Gamma \vdash \text{let } x \Leftarrow \text{get in put } (x + 1) : \mathbf{T} [\text{get } \mathbb{Z}, \text{put } \mathbb{Z}] ()$$

cf. session-typed π -calculus version

$$c : \oplus[\text{get} : ?[\mathbb{Z}]. \oplus[\text{put} : ![\mathbb{Z}].\text{end}]] \vdash \text{get}(c)(x).\text{put}(c)\langle x+1 \rangle.0$$
$$\approx [\text{get } \mathbb{Z}, \text{put } \mathbb{Z}]$$

Sessions as effects

- Effect handler process [e.g., variable agent]

[cf. Bauer, Pretnar “Programming with algebraic effects and handlers.”]

- Effect channel [a session channel for communicating with handler]

... whose session type is (encoding of) effect annotation

- “Threading” effect channel through control flow of encoding

[cf. state $\langle e, s \rangle \rightarrow \langle e', s' \rangle$ or monadic semantics $a \rightarrow M b$]

Sessions as effects

- Encoding of general effect sequential composition
- Parameterised, for a particular notion of effect by:
 - *Effect handler* [variable agent]
 - Interpretation of effects annotations into sessions

$$\llbracket [] \rrbracket = \text{end}$$

$$\llbracket (\text{get } \tau) : F \rrbracket = \oplus[\text{get} : ?\llbracket \tau \rrbracket. \llbracket F \rrbracket]$$

$$\llbracket (\text{put } \tau) : F \rrbracket = \oplus[\text{put} : !\llbracket \tau \rrbracket. \llbracket F \rrbracket]$$

- Encoding of operations [get, put]
 - using $\text{get}(c)(x).P = (c \triangleleft \text{get}).c?(x).P$ etc.

Embedding

$$\llbracket \Gamma \vdash M : \mathbf{T} \mathbf{F} \tau \rrbracket_r^{eff} = \llbracket \Gamma \rrbracket, r : !\llbracket \tau \rrbracket, eff : \llbracket \mathbf{F} \rrbracket \vdash \dots$$

$$\llbracket \Gamma \vdash \text{let } x \leftarrow M \text{ in } N : \mathbf{T} \mathbf{F} \tau \rrbracket_r^{eff} =$$
$$\nu s . (\llbracket M \rrbracket_s \mid \underline{s}?(x).\llbracket N \rrbracket_r)$$

 both using *eff*

Not well-typed wrt. sessions!

Embedding

$$\llbracket \Gamma \vdash M : \mathbf{T} \mathbf{F} \tau \rrbracket_r^{eff} = \llbracket \Gamma \rrbracket, r : !\llbracket \tau \rrbracket, eff : \llbracket \mathbf{F} \rrbracket \vdash \dots$$

$$\nu ei, eo . (\llbracket \Gamma \vdash M : \mathbf{T} \mathbf{F} \tau \rrbracket_r^{ei, eo} \mid \underline{ei}! \langle eff \rangle . eo(underline{eff}'))$$

$$\llbracket \Gamma \vdash M : \mathbf{T} \mathbf{F} \tau \rrbracket_r^{ei, eo} = \forall \mathbf{g} .$$

$$\llbracket \Gamma \rrbracket, r : !\llbracket \tau \rrbracket, ei : ?\llbracket \mathbf{F} \bullet \mathbf{g} \rrbracket, \underline{eo} : !\llbracket \mathbf{g} \rrbracket \vdash \dots$$

receive effect channel

send effect channel

$$\llbracket \Gamma \vdash \mathbf{run} M : \tau \rrbracket_r$$

$$= \nu eff . (\llbracket \Gamma \vdash M : \mathbf{T} \mathbf{F} \tau \rrbracket_r^{eff} \mid H(\underline{eff}))$$

Embedding

$$\langle \Gamma \vdash \langle M \rangle : \mathbf{T} \ \cancel{\tau} \ \rangle_r^{ei, eo} = ei?(c). \langle M \rangle_r . \underline{eo}! \langle c \rangle$$

where $\forall \mathbf{g} . \llbracket \Gamma \rrbracket ; r : !\llbracket \tau \rrbracket, ei : ?\llbracket \mathbf{g} \rrbracket, \underline{eo} : !\llbracket \mathbf{g} \rrbracket \vdash ei?(c). \langle M \rangle_r . \underline{eo}! \langle c \rangle$

$$\langle \Gamma \vdash \text{let } x \leftarrow M \text{ in } N : \mathbf{T} \ (\mathbf{F} \bullet \mathbf{G}) \ \tau \ \rangle_r^{ei, eo} = \\ \vee q, a . \left(\langle M \rangle_q^{ei, a} \mid \underline{q}?(x). \langle N \rangle_r^{a, eo} \right)$$

where $\forall \mathbf{h} . \quad q : !\llbracket \sigma \rrbracket, ei : ?\llbracket \mathbf{F} \bullet \mathbf{h} \rrbracket, \underline{a} : !\llbracket \mathbf{h} \rrbracket \vdash \langle M \rangle_q^{ei, a}$
 $\mathbf{h} \rightarrow \mathbf{G} \bullet \mathbf{h}'$

$\forall \mathbf{h}' . \quad x : \llbracket \sigma \rrbracket ; r : !\llbracket \tau \rrbracket, a : ?\llbracket \mathbf{G} \bullet \mathbf{h}' \rrbracket, \underline{eo} : !\llbracket \mathbf{h}' \rrbracket \vdash \langle N \rangle_r^{a, eo}$

Embedding

$$\langle \Gamma \vdash \langle M \rangle : \mathbf{T} \ \cancel{\tau} \ \rangle_r^{ei, eo} = ei?(c). \langle M \rangle_r . \underline{eo}! \langle c \rangle$$

where $\forall \mathbf{g} . \llbracket \Gamma \rrbracket ; r : !\llbracket \tau \rrbracket, ei : ?\llbracket \mathbf{g} \rrbracket, \underline{eo} : !\llbracket \mathbf{g} \rrbracket \vdash ei?(c). \langle M \rangle_r . \underline{eo}! \langle c \rangle$

$$\langle \Gamma \vdash \text{let } x \leftarrow M \text{ in } N : \mathbf{T} \ (\mathbf{F} \bullet \mathbf{G}) \ \tau \ \rangle_r^{ei, eo} =$$

$$\vee q, a . \left(\langle M \rangle_q^{ei, a} \mid \underline{q}?(x). \langle N \rangle_r^{a, eo} \right)$$

where $\forall \mathbf{h} . q : !\llbracket \sigma \rrbracket, ei : ?\llbracket \mathbf{F} \bullet \mathbf{G} \bullet \mathbf{h}' \rrbracket, \underline{a} : !\llbracket \mathbf{G} \bullet \mathbf{h}' \rrbracket \vdash \langle M \rangle_q^{ei, a}$
 $\mathbf{h} \rightarrow \mathbf{G} \bullet \mathbf{h}'$

$\forall \mathbf{h}' . x : \llbracket \sigma \rrbracket ; r : !\llbracket \tau \rrbracket, a : ?\llbracket \mathbf{G} \bullet \mathbf{h}' \rrbracket, \underline{eo} : !\llbracket \mathbf{h}' \rrbracket \vdash \langle N \rangle_r^{a, eo}$

Example

$\llbracket \Gamma \rrbracket, r : !\llbracket \text{int} \rrbracket, \text{eff} : \llbracket [\text{get } \mathbb{Z}, \text{put } \mathbb{Z}] \rrbracket \vdash$

$\llbracket \Gamma \vdash \text{let } x \leftarrow \text{get in put } (x + 1) : \mathbf{T} [\text{get } \mathbb{Z}, \text{put } \mathbb{Z}] \mathbb{Z} \rrbracket_r^{\text{eff}}$

$\llbracket \Gamma \vdash \text{run } M : \tau \rrbracket_r = \text{veff} . (\llbracket \Gamma \vdash M : \mathbf{T} \mathbf{F} \tau \rrbracket_r^{\text{eff}} \mid \text{Var}(\text{eff}, 0))$

Soundness

$$\Gamma \vdash M = N : \mathbf{T} \mathbf{F} \tau \implies$$

$$\llbracket \Gamma \rrbracket; (r : !\llbracket \tau \rrbracket).\mathbf{end}, e : \llbracket F \rrbracket) \vdash \llbracket M \rrbracket_r^e \approx \llbracket N \rrbracket_r^e$$

$$\mathbf{let} \ x \leftarrow M \ \mathbf{in} \ \langle x \rangle \ = \ M \quad \text{(left unit)}$$

$$\mathbf{let} \ x \leftarrow \langle v \rangle \ \mathbf{in} \ M \ = \ M[v/x] \quad \text{(right unit)}$$

$$\mathbf{let} \ x \leftarrow M \ \mathbf{in} \ (\mathbf{let} \ y \leftarrow N_1 \ \mathbf{in} \ N_2) \ = \quad \text{(associativity)}$$

$$\mathbf{let} \ y \leftarrow (\mathbf{let} \ x \leftarrow M \ \mathbf{in} \ N_1) \ \mathbf{in} \ N_2 \quad [if \ x \# N_1]$$

Completeness

$$\llbracket \Gamma \rrbracket, r : !\llbracket \tau \rrbracket, \mathit{eff} : \llbracket \mathbf{F} \rrbracket \vdash \llbracket M \rrbracket_r^{\mathit{eff}} \approx \llbracket N \rrbracket_r^{\mathit{eff}}$$

$$\implies \Gamma \vdash M \approx N : \mathbf{T} \mathbf{F} \tau$$

Application

Use session- π as intermediate language

- Effect-informed optimisations, e.g. implicit parallelism

if $\Gamma \vdash M : \mathbf{T} \ \emptyset \ \sigma$ and $\Gamma \vdash N : \mathbf{T} \ \mathbf{F} \ t$

then $(\text{let } x \leftarrow M \text{ in } (\text{let } y \leftarrow N \text{ in } P)) \Big|_r^{\text{ei, eo}}$
 $= \nu q, s, \text{eb}. (\llbracket M \rrbracket_q \mid (\llbracket N \rrbracket_s^{\text{ei, eb}} \mid \bar{q}?(x).\bar{s}?(y).(\llbracket P \rrbracket_r^{\text{eb, eo}}))$

- Semantics of *concurrent effects*
 - e.g., *non-interference, atomicity via sessions*

Effects as sessions (summary)

- Sessions and session types expressive enough to encode effects with a **causal** effect system
 - Per effect notion [e.g., state, counting, I/O]:
effect mapping, handler, encoding operations
- Extended to **case** and **fix** effect-control-flow operator
- Set-based effect systems recovered by transforming causal

Details see dorchard.co.uk:

“Using session types as an effect system” (Orchard, Yoshida, PLACES 2015)

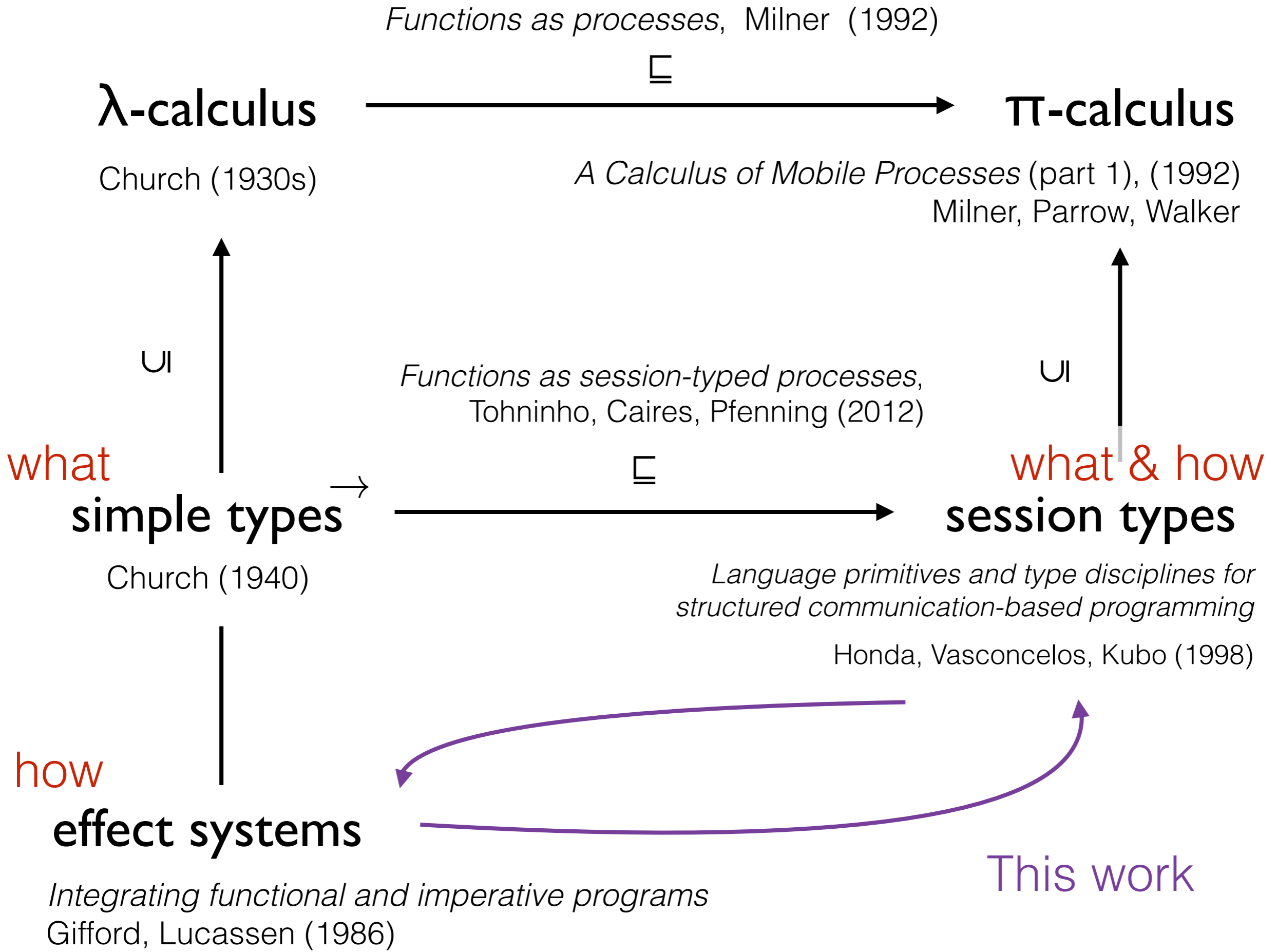
Effects as sessions

$$\llbracket \Gamma \vdash M : \mathsf{T} \mathbf{F} \tau \rrbracket_r^{eff} \longrightarrow \llbracket \Gamma \rrbracket, r : !\llbracket \tau \rrbracket, eff : \llbracket \mathbf{F} \rrbracket \vdash P$$

Sessions as effects?

$$\Gamma; \Delta \vdash P \longrightarrow \llbracket \Gamma \rrbracket \vdash M : \mathsf{T} \llbracket \Delta \rrbracket \text{ unit}$$

- Reuse existing effect-system approaches:
 - *Embedding effect systems in Haskell* (Orchard, Petricek, 2014)
- Session types for existing libraries (e.g., CloudHaskell)



Conclusion

Effects into sessions

- Shows the expressive power of session types
- Incorporate effect information into specifications (e.g. Scribble)
- Use pi-calculus as intermediate language

Sessions into effects (work in progress)

- Embed session types into existing languages
- Shows the expressive power of effect typing
 - “Type & Effect system: Behaviours for concurrency” Amtoft, Nielson, Nielson 1999

the tale of ~~two~~ ^{one} type systems ?

Thanks!!!

Other things I do...

Theory

- **Coeffects [dual to effects, contextual effects]** $\forall i. \Box_i$
 $\Box_{(i+j)} A \rightarrow \Box_i \Box_j A \quad \Box_0 A \rightarrow A \quad \Box_i(A \rightarrow B) \rightarrow \Box_i A \rightarrow \Box_i B$

[thesis,ICALP'13,ICFP'14] with Alan Mycroft, Tomas Petricek
[w.i.p] with Marco Gaboardi, Shinya Katsumata

- **Models/analysis of timed, communicating processes [for music!]**
[FARM'14,w.i.p] with Sam Aaron

Applications

- **Haskell type system and compiler**
[thesis,FLOPS'10,IFL'12,ICFP'13,Haskell'14]
with Alan Mycroft, Tomas Petricek, Neal Glew, Leaf Petersen, Tom Schrijvers
- **Languages and tools for computational science**
SSI Fellowship (2015) [WRT'13,ICCS'13,ICCS'14] with Andy Rice